### Brevia

# SHORT NOTES

# Computer-based method to separate heterogeneous sets of fault-slip data into sub-sets

QIN HUANG\*

Tectonique Quantitative, Département de Géotectonique, Université Pierre & Marie Curie, 4 place Jussieu, 75252 Paris Cédex 05, France

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Abstract—Based on the theory of dynamical cluster analysis, a procedure for separating heterogeneous sets of fault-slip data into homogeneous sub-sets is proposed, which enables the stress state associated with each sub-set to be determined separately within a data set. This procedure is suitable for any method of determining the principal stress axes using fault-slip data and runs on a micro-computer with graphical output.

### **INTRODUCTION**

At a site of observation, measurements of faults with their planar orientations and senses of slip shown by linear slickensides on faults are, sometimes, dispersed in spherical space. Such dispersion may represent two cases. Firstly, the set of fault-slip data may be composed of several dynamically homogeneous sub-sets which result from different single tectonic events. Secondly, the last tectonic event may have reactivated pre-existing fractures present at the site of observation. In both cases, such data sets give rise to difficulties in computation of the principal stress axes, because these axes are calculated by assuming that the whole data set resulted from a single stress state. In the first case, the different homogeneous sub-sets should be separated a priori and the corresponding stress axes for each sub-set computed independently. For the second case, up to now, it is difficult to say whether such data sets are suitable for reconstructing the paleo-stress axes, as McKenzie & Jackson (1983) stated that there is no obvious relation between the reactivated faults and the paleo-stress. Bott's oblique faulting model did not take into account any possible propagation and deformation of pre-existing fractures when they are subjected to later stress states (Bott 1959). It is prudent to use reactivated faults only for analysis of the chronology of faulting events, but not for reconstruction of stress states (Huang 1987).

This paper treats the first case stated above. For a heterogeneous data set, the theory of dynamical cluster analysis proposed by Diday (1971), which takes the distance between the elements in a population of data as a measure of similarity, is used to separate the data into sub-sets. As orientations of fault planes and striae are related to those of the principal stress axes, the separation procedure should be based on such a relation, and

\*Present address: 505 Yokoyama Mansion, Honkomagome 3-1-8, Bunkyo-Ku, Tokyo 113, Japan. faults in each separated sub-set should be homogeneous with respect to the orientations of associated stress axes. Many procedures for determining the stress axes related to a set of fault-slip data have been presented (Carey & Brunier 1974, Angelier 1979, Etchecopar et al. 1982, Michael 1984, Huang & Angelier 1987); each method has its own characteristics, but the simplest one is Huang & Angelier's vector analysis method (Huang & Angelier 1987). They replace the principal stress axes  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$ in Anderson's faulting model by axes  $P_{\theta}$ , B and  $T_{\theta}$  for each fault in a set of data as shown in Fig. 1. They then use the statistics of orientation data (Mardia 1972) to calculate the preferred axes for each of three groups of vectors and assign them as the principal stress axes associated with this fault data set. The present separation procedure is associated with this method, but it can also be applied with other procedures.

### PRINCIPLE

Consider a heterogeneous set of fault-slip data composed of M homogeneous sub-sets (this number can be



Fig. 1. Relations of conjugate faults, associated principal stress axes  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$ , axes  $P_{\theta}$ , B,  $T_{\theta}$  and dihedral angle  $2\theta$ ; axis B being perpendicular to the normal of fault plane and direction of striae; axis  $P_{\theta}$  being perpendicular to B and making angle  $\theta$  and  $\pi/2 - \theta$  with the normal to fault plane and striae, respectively; axis  $T_{\theta}$  being perpendicular to both axes  $P_{\theta}$  and B.

roughly estimated either by visual inspection on a stereonet of fault projections or by field observations). The principle of dynamical cluster analysis (Diday 1971) consists of comparisons of the distances between a given element (fault) with each of M sets of stress axes, each set of stress axes being defined by three orthogonal vectors  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$  (referred to as the nucleus hereafter). As a set of stress axes is determined by three orthogonal vectors and a fault can also be characterized by a set of three orthogonal vectors (axes  $P_{\theta}$ , B,  $T_{\theta}$ ), the distance, in our case, is the sum of three angular distances between two sets of vectors (each angular distance should be defined between corresponding axes, for example between  $P_{\theta}$  and  $\sigma_1$ , etc.). The comparison of distances between an element and M sets of stress axes will result in a classification of the element: the element belongs to a stress state for which the sum of the angular distances is minimum. Repeating this comparison for each fault, at the end of the comparison procedure (a cycle) each element has been temporarily classified and related to a stress state, we then get M sets of faults. We recalculate the principal stress axes for each of the M sets of faults and compare with previous ones. If the difference between 'old' and 'new' stress axes is negligible the separation procedure is then completed, otherwise, we replace 'old' sets of stress axes by 'new' ones, and the next cycle begins. This separation procedure is cyclical.

#### **METHOD**

For each fault, based on the model of Anderson (1942), one can easily determine the orientations of axes  $P_{\theta}$ , B and  $T_{\theta}$  knowing the normal to the fault plane, the sense and direction of slip and the dihedral angle  $2\theta$ , as shown in Fig. 1 (Huang 1987). Before the separation procedure begins, we have to specify initial nuclei for the M sets of stress axes. The first guess can be guided by geological information at the site of observation, a good guess leads to less iterations in the separation procedure, but a poor guess will not substantively change the results. In practice, the first nuclei can be chosen as M sets of axes  $P_{\theta}$ , B and  $T_{\theta}$  associated with M faults, but they should be rather dispersed.

Let  $N_j$  (including three orthogonal vectors  $N_{j1}$ ,  $N_{j2}$ ,  $N_{j3}$ , j = 1, 2, ..., M) be the *j*th set of stress axes (nucleus) and define the sum of distances  $(D_j)$  between an element and a nucleus as:

$$D_{j} = |P_{\theta} \cdot N_{j1}| + |B \cdot N_{j2}| + |T_{\theta} \cdot N_{j3}|,$$

the right-hand side of the equation representing the absolute value of the scalar product of two vectors. The first iterative cycle begins by calculating the sum of distances between an element and each nucleus, the fault being assigned to the nucleus which produces a maximum value of  $D_j$  (i.e. minimum angular distance). Iteration continues by similar processing of the other fault measurements in the data set. By the end of the cycle, each fault has been assigned to a nucleus, and we have M groups of faults temporarily classified. After

calculating separately the principal stress axes for each group of faults, using Huang & Angelier's method (or other methods), we get M sets of new stress axes (new nuclei). If the angles between corresponding 'old' and 'new' nuclei are greater than the 'test angle' set by the user (2° in the case studied), the separation procedure has not stabilized. In this case, the 'new' nuclei estimated at the end of a cycle replace the 'old' ones and the next cycle begins. Eventually the procedure stabilizes if the number of sub-sets (M) is suitably chosen. If M is too small the separation of the data set will not be complete; if M is too large some of the resulting sets of stress axes will be very close to each other.

Another procedure, following the separation of stress axes, consists of picking out 'bad' fault measurements from the data set before going into final computation of the principal stress axes associated with each sub-set. Taking 45° as a limit, for each fault, if the sum of angles between axes  $P_{\theta}$ , B and  $T_{\theta}$  and the final nucleus (stress axes) is greater than this limit the faults can be rejected. This operation enables the quality of computed stress axes to be improved. The limiting angle should be reasonable according to the data quality: where the data are well clustered, the angle can be small and vice versa.

## APPLICATION AND DISCUSSIONS

Figure 2 shows an example collected from the Sisteron area, SE France. The fault data are rather heterogeneous with sinistral and dextral strike-slip faults occur in similar directions (Fig. 2a). By visual inspection, there may exist at least two sets and these have been separated into two sets of strike-slip faults at the end of six cycles with the test angle being 2°. Their associated stress axes are computed using the method of Huang & Angelier (1987). Three faults were eliminated since they have angles greater than 45° to the two stress systems determined. In reality, these three faults result from other tectonic events related to a compression with maximum stress trending about N040°E. This tectonic event is identified in other sites of observation (Huang 1987). It is important to examine the faults picked out from the data set since they may indicate other stress states (Fig. 2f).

In the method of Huang & Angelier (1987), the dihedral angle  $2\theta$  is required for determining the orientation of axes  $P_{\theta}$ , B and  $T_{\theta}$ . This angle is generally estimated by averaging all measurements on pairs of conjugate faults observed in the field (60° in the case studied). The angle does not affect the results when both sets of conjugate faults are present in the data set, but where only one set is present it plays a very important role in determining the stress axes (Huang 1987).

The separation procedure presented above is rather simple and can be used with any kind of algorithm which determines the stress axes. It runs on an IBM-PC with 256 RAM. For the presented data set, whole separation needed 1 min (with Huang & Angelier's method for determining the stress axes).



Fig. 2. Results of separation procedure applied to faults from the Sisteron area, SE France. (a) Twenty-six strike-slip faults with their planes and striae; (b) first set of homogeneous faults with orientations of the principal stress axes (shown by the black arrows); (c) axes  $P_{\theta}$ , B,  $T_{\theta}$  associated with the faults in (b); (d) second set of homogeneous faults with the principal stress axes; (e) axes  $P_{\theta}$ , B,  $T_{\theta}$  associated with the faults in (d); (f) faults picked out from the data set corresponding to a compression trending near N040°E.

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#### REFERENCES

- Anderson, E. M. 1942. *The Dynamics of Faulting*. Oliver & Boyd. Edinburgh.
- Angelier, J. 1979. Determination of the mean principal directions of stresses for a given fault population. *Tectonophysics* 56, T17-T26.
- Bott, M. H. P. 1959. The mechanisms of oblique slip faulting. Geol. Mag. 96, 109-117.
- Carey, E. & Brunier, B. 1974. Analyses théorique et numérique d'un modèle mécanique élémentaire appliqué à l'étude d'une population de failles. C.r. hebd. Seanc. Acad. Sci., Paris 290(D), 297–300.
- Diday, E. 1971. Une nouvelle méthode de classification automatique

et reconnaissance des formes: la méthode nuées dynamiques. Rev. Stat. Appl. 19, 283-300.

- Etchecopar, A., Vasseur, G. & Daignières, M. 1982. An inverse problem in microtectonics for the determination of stress tensor from fault striation analysis. J. Struct. Geol. 3, 51-65.
- Huang, Q. 1987. Analyse géometrique et dynamique de la fracturation: application a la région de Sisteron et au Golfe de Suez. Mém. Sci. de la Terre. 87-14, Université Paris VI.
  Huang, Q. & Angelier, J. 1987. Les systèmes de failles conjuguées:
- Huang, Q. & Angelier, J. 1987. Les systèmes de failles conjuguées: Une méthode d'identification, de séparation et de calcul des axes de contrainte. C.r. hebd. Seanc. Acad. Sci., Paris. 304, 465–468.
- Mardia, K. V. 1972. *Statistics of Directional Data*. Academic Press, London and New York.
- McKenzie, D. & Jackson, J. 1983. The relationship between strain rate, crustal thickening, paleomagnetism, finite strain and fault movements within a deformation zone. *Earth Planet. Sci. Lett.* **65**, 182–202.
- Michael, A. J. 1984. Determination of stress from slip data: faults and folds. J. geophys. Res. 81, 11,517–11,526.